The Niels Henrik Abel mathematics competition: Second round 2017–2018

11 January 2018 (English)

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Do not turn the page until told to by your teacher!

The second round of the Abel competition consists of 10 problems to be solved in 100 minutes. The solutions are integers between 0 and 999, inclusive. Write your answers in the lower left hand side of the form.

You get 10 points for each correct answer and 0 points for a blank or wrong answer. This yields a total between 0 and 100 points.

No aids other than scratch paper and writing implements (including compass and ruler, but not protractor) are allowed.

When your teacher says so, you can turn over the page and begin working on the problems.

Fill in using block letters

Name			Date of birth
Address			Gender F ☐ M ☐
Post code	Post office		
School			Class
Citizenship	Email		Mobile phone
☐	Check the box to allow us to put your name on the results list. (Regardless, we only publish results for the best third.)		

Answers

1		6	
2		7	
3		8	
4		9	
5		10	

For the teacher

Correct: ☐ $\cdot 10 =$ ☐

Problem 1

In the triangle ABC, two sides are $AB = AC = 720$, and $\angle A = 90°$. The midpoint of side BC is E, and the midpoint of segment AE is F. The line through C and F meets AB at D. How long is the line segment AD?

Problem 2

What is the smallest positive integer having no prime factor in common with the product $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot \cdots \cdot 199 \cdot 200$?

Problem 3

A positive integer is called *geometric* if the digits are different and form a geometric sequence. What is the difference between the largest and the smallest three digit geometric integer?

Problem 4

What is the sum of all possible values for $60x/y$, where x and y are pairs of values which satisfy the equations $1 - 2x + 3y - 4x^2 + 5xy + 6y^2 = 0$ and $x - y = 1$?

Problem 5

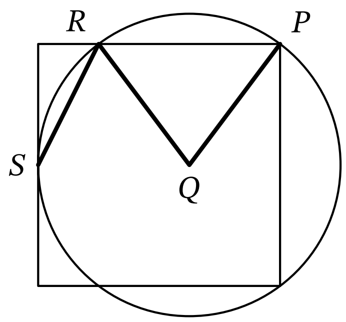
A rectangle composed of whole squares on a chess board is called *imbalanced* if it covers a different number of black and white squares. How many imbalanced rectangles exist on a regular (8×8) chess board?

Problem 6

How many seven digit numbers can be created by permuting the digits of 1234567 so that each odd digit is next to exactly one other odd digit?

Problem 7

A circle centred at Q passes through two adjacent corners of a square with side length 32, and is tangent to the opposite side. What is the length of the path $PQRS$ in the picture, rounded to the nearest integer?



Problem 8

What is the value of $p(1)+p(7)+p(19)+p(25)$, if $p(x)=x^3-39x^2+507x-2018$?

Problem 9

Which positive integer less than 1000 has the most divisors?

Problem 10

How many binary strings are there of length eleven, which do not contain more than two consecutive zeroes? (Example: 00111101001.)

Solutions are posted on 12 January at 17.00 on

abelkonkurransen.no