The Niels Henrik Abel mathematics competition
2015–2016

First round   5 November 2015

Do not turn the page until told to by your teacher!

The first round of the Abel competition consists of 20 multiple choice problems to be solved in 100 minutes. Only one of the five alternatives is correct. Write your answers in the lower left hand side of the form.

You get 5 points for each correct answer, 1 point for a blank answer, and 0 points for a wrong answer. This yields a total between 0 and 100 points. A totally blank response results in 20 points.

No aids other than scratch paper and writing implements (including compass and ruler) are allowed.

When your teacher says so, you can turn over the page and begin working on the problems.

Fill in using block letters

Name		Date of birth
Address		Gender F ☐ M ☐
Post code	Post office	
School		Class
Have you participated in the Abel competition before? If so, what year(s)?		

Answers

1 ☐	11 ☐
2 ☐	12 ☐
3 ☐	13 ☐
4 ☐	14 ☐
5 ☐	15 ☐
6 ☐	16 ☐
7 ☐	17 ☐
8 ☐	18 ☐
9 ☐	19 ☐
10 ☐	20 ☐

For the teacher

Correct: ☐ · 5 = ☐
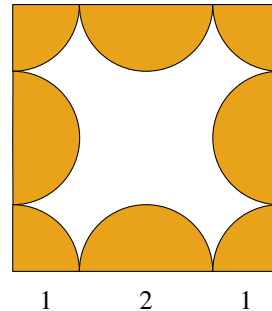
Unanswered: ☐ + ☐

Points: = ☐

Problem 1

Anne and Beate together have kr 120, Beate and Cecilie together have kr 60, and Anne and Cecilie together have kr 70. How many kroner do they have in total?

A 120   B 125   C 130   D 180   E 190

Problem 2

What is the area of the shaded region in the picture?

A 3π B 5π C 6π D 9π E 12π

1   2   1

Problem 3

Which one of the following numbers is equal to $4^7 \cdot 2^4$?

A 8^3 B 8^6 C 8^{11} D 8^{14} E 8^{28}

Problem 4

The numbers a_1, a_2, a_3, and a_4 are drawn at random from the set $\{0, 1, 2, \dots, 9\}$. We allow the same number to be drawn repeatedly. What is the probability that $a_1 a_4 - a_2 a_3$ is an even number?

A $\dfrac{1}{2}$ B $\dfrac{1}{4}$ C $\dfrac{3}{8}$ D $\dfrac{3}{4}$ E $\dfrac{5}{8}$
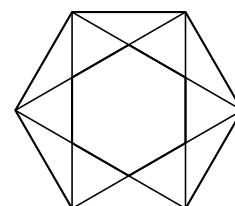
Problem 5

Which one of the following numbers is equal to $\dfrac{2016^4 - 2015^4}{2015^2 + 2016^2}$?

A 2015   B 4031   C 4033   D $2 \cdot (2016^2 - 2015^2)$ E $2015 \cdot 2016$

Problem 6

There are two regular hexagons in the picture. What is the ratio of the area of the larger one to that of the smaller one?

A 2   B 3   C $2\sqrt{3}$ D 4   E None of these

Problem 7

Idun is rolling four ordinary six-faced dice, with the faces labelled 1 through 6. What is the probability that her total score is divisible by 3?

A $\dfrac{71}{6^3}$ B $\dfrac{11}{36}$ C $\dfrac{1}{3}$ D $\dfrac{1}{7}$ E $\dfrac{1}{6}$

Problem 8

How many of the integers 0, 1, 2, …, 999 are neither divisible by 9 nor contain the digit 9?

A 486   B 487   C 512   D 648   E 649

Problem 9

The sum of Anne's and Berit's ages is 60 years. Anne is three times as old as Berit was when Anne was the age that Berit is now. What is the sum of the digits of Anne's age?

A 1   B 3   C 5   D 7   E 9

Problem 10

Three points A, B, and C in the plane have coordinates $(0, 4)$, $(6, 2)$, and $(10, 4)$, respectively. Then $\angle ABC$ equals

A 105°   B 120°   C 135°   D 145°   E None of these

Problem 11

A divisor of an integer N is an integer which divides evenly into N. Both 1 and N are counted among the divisors of N. The number of positive integers less than 100 having exactly three divisors is

A 2   B 3   C 4   D 5   E 6

Problem 12

You have two identical decks of cards. You remove the four "eight" cards from one deck and add them to the other. Then you draw one card at random from each deck. What is the probability that the cards you draw constitute a pair? (A pair is two cards of the same rank, not necessarily the same suit. The 52 cards in a deck of cards have all possible combinations of 4 suits and 13 ranks.)

A $\dfrac{1}{12}$ B $\dfrac{1}{13}$ C $\dfrac{1}{14}$ D $\dfrac{3}{56}$ E $\dfrac{7}{56}$
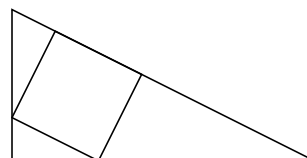
Problem 13

Which one of the following numbers is largest?

A $2014 \cdot 2016$ B $1971 \cdot 2060$ C $2000^2 + 15^2$ D $41{,}5^2 \cdot 48{,}5^2$
E 2015^2

Problem 14

The large triangle in the picture is a right triangle whose short sides have lengths 1 and 2. What is the area of the inscribed square?

A $\dfrac{5}{13}$ B $\dfrac{2}{5}$ C $\dfrac{34}{81}$ D $\dfrac{20}{49}$ E $\dfrac{7}{16}$

Problem 15

How many positive integers m have the property that $m^2 + 2015$ is a perfect square?

A 2   B 4   C 6   D 8   E 10

Problem 16

It is possible to select 15 children from a school with 2015 children in N different ways. What is the last digit of N?

A 0   B 2   C 4   D 8   E None of these

Problem 17

What is the sum of the digits of the smallest positive integer n such that $n^4 + 6n^3 + 11n^2 + 6n$ is divisible by 700?

A 5   B 7   C 8   D 10   E 12

Problem 18

The triangle ABC has two equal sides $AB = AC = 1$, and $\angle CAB = 135°$. The circle S is centred at A, and is tangent to BC. What is the area of S?

A $\dfrac{3}{20}\pi$ B $\dfrac{\sqrt{2}}{8}\pi$ C $\dfrac{2-\sqrt{2}}{4}\pi$ D $\dfrac{1+\sqrt{3}}{16}\pi$ E $\dfrac{\sqrt{35}}{40}\pi$

Problem 19

The set A_0 is $\{1, 2, 3, 4\}$. For $i = 0, 1, 2, \ldots$, the set A_{i+1} consists of all possible sums you can get by adding two different members of A_i. How many different numbers are there in the set A_{10}?

A 512   B 515   C 1024   D 1027   E 3073

Problem 20

The number x is given by

$$\frac{1}{x} = \frac{1}{2016^2} + \frac{1}{2017^2} + \cdots + \frac{1}{4030^2}.$$

Which of the following numbers is nearest to x?

A 2015   B 2016   C 3024   D 4029   E 4031

The solutions are published on 6 November at 17:00 on

abelkonkurransen.no