

Second round

Problem 1

What is the largest prime factor in 899?

Problem 2

In the figure there are 9 rectangles. How many rectangles may be found in the figure to the right?



Problem 3

When multiplying out the expression

$$\frac{(1+x)(2+x^2)(3+x^3)\cdots(103+x^{103})}{1\cdot 2\cdot 3\cdots 103},$$

we obtain a polynomial of the form $a_0 + a_1x + \ldots + a_{5356}x^{5356}$. What is the sum of its coefficients, $a_0 + a_1 + \cdots + a_{5356}$?

Problem 4

How many of the permutations of 1, 2, 3, 4, 5, 6 are such that each odd number is next to at least one even number?

Problem 5

In the rectangle ABCD, the side AB is of length 120 and the side BC of length 240. Let E be the midpoint of the segment BC, and let the points F and G lie on the segments AE and DE, respectively, such that the line FG is parallel to the line AD and the area of the triangle FEG is half the area of the triangle AED.

What is the length of the segment EF?

Problem 6

How many positive integers less than 2013 are divisible by none of 2, 3, 4 and 5?



Problem 7

Two points O and P lie a distance OP = 40 apart. A circle of radius $10\sqrt{7}$ is centered at O, and a circle of radius 30 is centered at P. The two circles intersect in two points A and B. A point C is located such that AC is a diameter in the smaller circle. What is the distance BC?

Problem 8

The sequence x_0, x_1, \ldots is defined through $x_0 = 3, x_1 = 18$ and $x_{n+2} = 6x_{n+1} - 9x_n$ for $n = 0, 1, 2, \ldots$. What is the smallest k such that x_k is divisible by 2013?

Problem 9

An octahedron is one of the Platonic solids. Its surface consists of eight equilateral triangles. If V denotes the volume of an octahedron in which the distance between two neighbouring vertices is $\sqrt{6}$, what is V^2 ?



Problem 10

Positive integers a, b, and c are such that 4abc + 2ab + 2bc + 2ca + a + b + c = 1006. What is a + b + c?

Solutions are posted on 18 January at 17.00 on abelkonkurransen.no