

The Abel competition 2012-2013
Second round
English
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## Problem 1

What is the largest prime factor in 899 ?

## Problem 2

In the figure
 there are 9 rectangles. How many rectangles may be found in the figure to the right?


## Problem 3

When multiplying out the expression

$$
\frac{(1+x)\left(2+x^{2}\right)\left(3+x^{3}\right) \cdots \cdots\left(103+x^{103}\right)}{1 \cdot 2 \cdot 3 \cdots \cdots 103}
$$

we obtain a polynomial of the form $a_{0}+a_{1} x+\ldots+a_{5356} x^{5356}$. What is the sum of its coefficients, $a_{0}+a_{1}+\cdots+a_{5356}$ ?

## Problem 4

How many of the permutations of $1,2,3,4,5,6$ are such that each odd number is next to at least one even number?

## Problem 5

In the rectangle $A B C D$, the side $A B$ is of length 120 and the side $B C$ of length 240. Let $E$ be the midpoint of the segment $B C$, and let the points $F$ and $G$ lie on the segments $A E$ and $D E$, respectively, such that the line $F G$ is parallel to the line $A D$ and the area of the triangle $F E G$ is half the area of the triangle $A E D$.

What is the length of the segment $E F$ ?

## Problem 6

How many positive integers less than 2013 are divisible by none of $2,3,4$ and 5 ?


## Problem 7

Two points $O$ and $P$ lie a distance $O P=40$ apart. A circle of radius $10 \sqrt{7}$ is centered at $O$, and a circle of radius 30 is centered at $P$. The two circles intersect in two points $A$ and $B$. A point $C$ is located such that $A C$ is a diameter in the smaller circle. What is the distance $B C$ ?

## Problem 8

The sequence $x_{0}, x_{1}, \ldots$ is defined through $x_{0}=3, x_{1}=18$ and $x_{n+2}=$ $6 x_{n+1}-9 x_{n}$ for $n=0,1,2, \ldots$. What is the smallest $k$ such that $x_{k}$ is divisible by 2013 ?

## Problem 9

An octahedron is one of the Platonic solids. Its surface consists of eight equilateral triangles. If $V$ denotes the volume of an octahedron in which the distance between two neighbouring vertices is $\sqrt{6}$, what is $V^{2}$ ?


Problem 10
Positive integers $a, b$, and $c$ are such that $4 a b c+2 a b+2 b c+2 c a+a+b+c=$ 1006. What is $a+b+c$ ?

