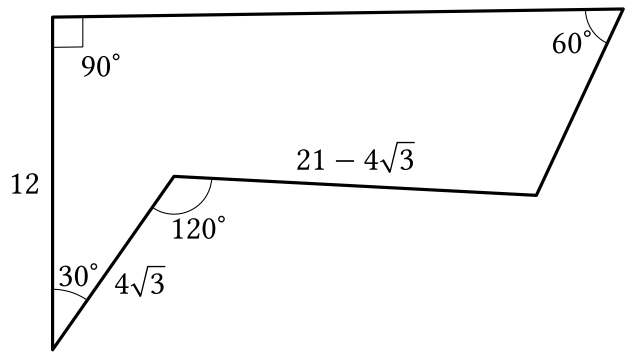
Problem 1

The final round of a mathematics contest consists of four problems, each picked from one of the topics A, G, K and T. All of the four topics are represented. Andrea bets that she can guess the order of the topics before the contest is held, and Beate bets against. They agree that Beate will buy Andrea n ice creams if Andrea's guess is correct, in exchange for Andrea buying one ice cream if Andrea is wrong. Assuming the game is fair, what is n?

Problem 2

How large is the area of the figure? (*Note that angles and ratios between side lenghts are not correct in the figure*! All you have to go on, are the measures given in the figure.)



Problem 3

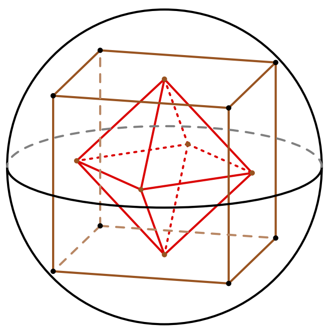
What is the smallest value of $a + b$, where a and b are positive integers with $2021 + a^2 = b^2$?

Problem 4

How many third degree polynomials P exist, all of whose coefficients are integers greater than 0 and less than 1000, so that $P(-1) = P(-2) = P(-3) = 0$?

Problem 5

All the corners of a cube lie on the surface of a ball with radius $2^{2/3} \cdot 3^{3/2}$. An octahedron has its corners in the centres of the six faces of the cube. What is the volume of this octahedron?



Problem 6

In how many ways can you colour all of the white squares in a 4×4 chess board with red, green and blue, so that any two squares meeting at a corner get different colours?

Problem 7

How many solutions does the equation

$$\tan(x)\sin(38^\circ) = \sin(52^\circ)$$

have with $2020 \le x \le 4040$, where x is measured in degrees?

Problem 8

What is the greatest integer $N < 1000$ so that N has exactly four divisors, while $N+1$ has an odd number of divisors? A divisor of N is an integer d with $1 \le d \le N$ dividing N.

Problem 9

What is the largest possible value of $\frac{1}{2}(a+b)$, with a and b being positive integers so that $(a+b)^2 = 2020\,a$?

Problem 10

Positive integers $n_2, n_3, n_4, \dots, n_{2020}$ satisfy

$$2^{n_2} + 3^{n_3} + 4^{n_4} + \cdots + 2020^{n_{2020}} \le \frac{2020 \cdot 2021}{2} + 47.$$

How many different values can $n_2 + n_3 + n_4 + \cdots + n_{2020}$ have?